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# Low-Energy Effective Lagrangian in Unified Theories with Non-Universal Supersymmetry Breaking Terms\*

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## Abstract

Supersymmetric grand unified theories with non-universal soft supersymmetry breaking terms are studied. By integrating out the superheavy fields at a unification scale, we compute their low-energy effective Lagrangian. We find new contributions to the scalar potential specific to the non-universal supersymmetry breaking.  $D$ -term contribution to the scalar masses is one example. The gauge hierarchy achieved by a fine-tuning in the superpotential would be violated in general due to the non-universal SUSY breaking terms. We show, however, it is preserved for a certain class of the soft terms derived from a *hidden* ansatz. We also discuss some phenomenological implications of the non-universal supersymmetry breaking, including predictions of the radiative electroweak symmetry breaking scenario and of no-scale type models.

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# 1 Introduction

Supersymmetry (SUSY) has been regarded as a beautiful mechanism which ensures the stability of the hierarchy, between the weak scale and the grand-unification (GUT) [1] or Planck scale against radiative corrections. There is, however, another virtue of SUSY which is not often quoted: it is that the SUSY breaking terms at the weak scale directly reflect the physics at very high energies thanks to the moderate renormalization effects, *i.e.*, the absence of the quadratic divergences. For instance, one can study experimentally whether the weak scale SUSY spectrum is consistent with GUT, by measuring the gaugino masses [2]. Also, the scalar mass spectrum has certain “sum rules” specific to symmetry breaking patterns [3, 4]. Therefore, it is important to know the predictions of the various models on the SUSY breaking terms at low energy.

Many efforts have been devoted to the low-energy predictions under the assumption of the universal SUSY breaking terms, which led to remarkable progress in recent years [5, 6]. The idea of the universal SUSY breaking terms was led by the non-observation of the large flavor changing neutral current processes due to the SUSY particle loops [7]. And the hidden sector SUSY breaking [8] combined with the minimal supergravity indeed leads to the universal SUSY breaking terms, with or without grand unification [9]. However a decade after these works, there are increasing interests in the non-universal form of the SUSY breaking terms, at least due to the following two reasons. (1) The unification scale is now believed to be substantially lower than the gravitational scale  $M_{Pl}/\sqrt{8\pi}$ ,<sup>1</sup> and the radiative correction changes the form of the supersymmetry breaking terms. (2) The superstring theory implies highly non-universal form of the Kähler potential in general [10]. Therefore, it is an important task to see what low-energy theory results from the unified theories with non-universal form of the supersymmetry breaking terms.

There are some indications that the non-minimal SUSY breaking terms give potentially important consequences in the low-energy effective Lagrangian. First, they give rise to the so-called  $D$ -term contribution to the scalar masses when the rank of the gauge group is reduced [11]. We pointed out in the previous letter [3] that they can be generated even at the GUT scale and give observable consequences on the weak scale scalar mass spectrum which is important to distinguish the various symmetry breaking patterns [3, 4]. Second, the non-universal SUSY breaking terms may ruin the fine-tuning in SUSY-GUT models that the Higgs doublet may acquire a mass of intermediate scale in general. Third, the non-minimal initial conditions at the GUT scale modify the phenomenological conclusions made in the literature, in predictions on neutralino cosmic abundance, radiative breaking scenario and so on.

In this paper, we derive the low-energy effective Lagrangian starting from the grand unified theories with non-universal SUSY breaking terms. In fact, we find new contributions to the SUSY breaking terms at the low energy, after integrating out the superheavy fields at the GUT scale. It of course reduces to that obtained in Ref. [9] in the case with the universal SUSY breaking terms. Furthermore, the non-minimality of the SUSY break-

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<sup>1</sup>Note that the difference between the GUT scale  $\simeq 10^{16}$  GeV and the gravitational scale  $\simeq 10^{18}$  GeV is one seventh of that between the GUT scale and the weak scale if measured in log-scale.

ing terms at the GUT scale can lead to many interesting phenomenological consequences. We point out, for instance, that the scalar lepton becomes always heavier than the bino due to the radiative correction between Planck and GUT scales, and the upper bound on the slepton mass in the no-scale type models [12, 13] becomes invalid. Possible effects on radiative breaking scenario are also discussed.

The paper is organized as follows. In section 2, we first review the low-energy Lagrangian with universal SUSY breaking terms. Then we point out the importance of studying the case of the non-universal SUSY breaking terms, and demonstrate its important outcomes in explicit examples. We give the low-energy effective Lagrangian with the non-universal SUSY breaking terms in section 3. Here we also give an ansatz of the SUSY breaking terms based on the assumption that the hidden sector is hidden; this ansatz is stable under the renormalization effects, and we show that the fine-tuning in SUSY-GUT is not ruined in this ansatz of the SUSY breaking terms. We point out phenomenological implications of the non-universal SUSY breaking terms in section 4. Section 5 is devoted to conclusions.

## 2 Why Non-universal Soft SUSY Breaking Terms?

In this section, we briefly explain the typical effects of the non-universal soft SUSY breaking terms, to demonstrate the potential importance of their low-energy consequences. First we review the basic results by Hall et al. [9], where the low-energy effective Lagrangian is derived in the case with the universal SUSY breaking terms. We then point out that the universal form of the SUSY breaking terms is not preserved by the radiative corrections. As examples of potential importance of the non-universal SUSY breaking terms, we show that the non-universal scalar mass of the superheavy fields gives rise to  $D$ -term contributions to the scalar masses in the low-energy effective Lagrangian. We also give an example that the non-universal soft SUSY breaking terms destabilizes the hierarchy between the unification and weak scales. These observations give us a motivation to study the low-energy consequence of the non-universal SUSY breaking terms in a more general framework.

### 2.1 Minimal Supergravity

The low-energy effective Lagrangian of the minimal supergravity was shown to be extremely simple in Ref. [9]. Though the discussion in that paper was based on the supergravity Lagrangian, we re-phrase their result in the flat limit, *i.e.*, in the context of the global SUSY Lagrangian with soft SUSY breaking terms.<sup>2</sup>

The minimal supergravity suggests the following form of the scalar potential in the observable sector,

$$V = V_{SUSY} + V_{\text{soft}}, \quad (2.1)$$

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<sup>2</sup>This treatment can be actually justified from their analysis, that the fields in the hidden sector do not shift as the light fields are varied at  $O(m_S)$ .

$$V_{SUSY} = -F^\kappa F_\kappa^* - \frac{1}{2} D^\alpha D_\alpha + F^\kappa \frac{\partial W}{\partial z^\kappa} + h.c. + g_\alpha D^\alpha z_\kappa^* (T^\alpha)_\lambda{}^\kappa z^\lambda, \quad (2.2)$$

$$V_{SUSY} = AW + Bz^\kappa \frac{\partial W}{\partial z^\kappa} + h.c. + |B|^2 z_\kappa^* z^\kappa, \quad (2.3)$$

where  $W$  is the superpotential,  $z^\kappa$  are the scalar fields, and  $A, B$  are soft SUSY breaking parameters.<sup>3</sup> This form of the SUSY breaking terms is referred to as “universal,” because all the scalar masses are equal.

The main result in Ref. [9] is that the following form of the SUSY breaking terms results in the low-energy Lagrangian after integrating out the superheavy fields in the above Lagrangian,

$$V_{SUSY} = -2AW_{eff} + (A + B)z^k \frac{\partial W_{eff}}{\partial z^k} + h.c. + |B|^2 z_k^* z^k, \quad (2.4)$$

and it still has the same form as the original one by suitable redefinition of the  $A$  and  $B$  parameters except the mass squared terms. Here  $z^k$  are the light scalar fields and  $W_{eff}$  is the superpotential  $W$  with the extremum values for superheavy fields plugged in. Moreover, the scalar mass terms are still universal with the same mass,  $B$ . It is noteworthy that the authors of Ref. [9] did the analysis including the hidden sector fields, and proved that they do not shift when the light fields fluctuate at  $O(m_S)$  where  $m_S$  is the SUSY breaking scale  $\sim 1$  TeV, while the constant term in the superpotential should be shifted to cancel the cosmological constant.

The remarkable simplicity of the low-energy Lagrangian led to a number of strong conclusions, like the natural absence of the flavor changing neutral currents [7] or radiative breaking scenario due to the heavy top quark [14]. Due to these successes, it became like a dogma in the phenomenological analysis of the SUSY models. However, it becomes increasingly apparent that the supergravity Lagrangian may not have the minimal form, and it may lead to important consequences on the low-energy effective Lagrangian, as will be discussed in the next subsections.

## 2.2 Naturalness of the Universal SUSY Breaking Terms

It was pointed out that the higher order corrections in general destroy the minimal form of the Kähler potential [15, 16]. This poses a question on the naturalness of the minimal supergravity Lagrangian. However, this discussion is based on the one-loop corrections with a naive cut-off set at the Planck scale, and such higher order corrections may be absent in specific dynamics beyond the Planck scale. Note that even when the Planck scale dynamics satisfies certain symmetry to ensure the minimal form of the SUSY breaking terms, we still expect that it will be modified by the radiative corrections below the Planck scale.

Let us give an example of the minimal  $SU(5)$  model with vanishing scalar masses at the Planck scale [17]. The scalar fields acquire their masses through renormalization

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<sup>3</sup>The definition here is related to that in Ref. [9] by  $m_g = B^*$  and  $m'_g = (A + 3B)^*$ . It is noteworthy that they also discussed a modification of the minimal supergravity based on  $U(n)$  invariance in the kinetic function to ensure the universality of the scalar masses.

group equations, which are different for  $\bar{\mathbf{5}}$  and  $\mathbf{10}$  representations. Their masses at the  $SU(5)$  breaking scale  $\simeq 2 \times 10^{16}$  GeV are

$$m_{\bar{\mathbf{5}}}^2 = 0.30M^2 \quad (2.5)$$

$$m_{\mathbf{10}}^2 = 0.45M^2, \quad (2.6)$$

where  $M$  refers to the  $SU(5)$  gaugino mass at the supergravity scale  $M_{Pl}/\sqrt{8\pi}$ . Apparently these contributions cannot be neglected in the phenomenological analyses, and are also non-universal. They become even larger in the non-minimal GUT models because the gauge coupling constant tends to be larger than that in the minimal  $SU(5)$  model. This demonstrates that the running between the Planck scale and the  $SU(5)$  breaking scale is not negligible [18]. Then the SUSY breaking terms do not have a minimal form any more at the GUT scale, where the superheavy fields should be integrated out.

Instead of the universal SUSY breaking terms, we will take the following ansatz for the SUSY breaking terms

$$V_{SUSY} = m_S A W + m_S B^\kappa(z) \frac{\partial W}{\partial z^\kappa} + h.c. + O(m_S^2) \text{ terms.} \quad (2.7)$$

(See section 3.1.2 for the notation.)<sup>4</sup> It will be shown that this form of the SUSY breaking terms is stable under renormalization, or in other words, natural in the weak sense. Since the minimal form is not stable under the renormalization, we believe this is the framework to work out the low-energy effective Lagrangian. Moreover, from the view point of supergravity, this form is the most general form of the SUSY breaking terms induced by superhiggs mechanism with the assumption that the hidden sector is ‘hidden’, *i.e.*, that the superpotential is a sum of two independent pieces consisting of observable and hidden fields, respectively. We leave the detail to section 3.1.2.

Recall, also, that the superstring theory suggests non-minimal forms of the SUSY breaking terms in general [19]. This is especially true when the SUSY is broken by the  $F$ -component of the moduli fields. The scalar masses depend on their modular weights. The gaugino masses are also non-universal.

These observations give us a strong motivation to study the low-energy effective Lagrangian from the unified theories with non-universal SUSY breaking terms. In the next two subsections, we will point out potentially important effects of the non-universality.

One remark on the squark degeneracy is in order. It has been often stated that one needs high degeneracy of the scalar masses to ensure the potentially large contribution of the squark loop diagrams to the  $K^0$ - $\bar{K}^0$  mixing. However, this does not require the degeneracy of all scalar masses at the GUT or Planck scale. The only requirement is the degeneracy of the first- and second-generation squarks with the same quantum numbers. Note that neither the  $D$ -term contributions nor the renormalization group evolution due to the gauge interactions destroy the degeneracy as far as they do not distinguish the generations of the light quarks. One may have highly degenerate squark masses due to

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<sup>4</sup> $B^\kappa$  are holomorphic functions of  $z^\lambda$  independent for each  $\kappa$ , and *not* the derivative of a single function  $B$ .

the gluino mass contribution even with the different masses as the initial conditions at the Planck scale. Though the flavor changing neutral current still puts strong constraint on the non-minimality, it does not diminish our interest to study its low-energy consequences.

## 2.3 Possible $D$ -term Contributions

Though the low-energy Lagrangian is surprisingly simple when it had universal form at the unification scale, there arise different contributions to  $V_{\text{SUSY}}$  when one integrates out the superheavy fields from the Lagrangian with non-universal form of the SUSY breaking terms. This has an important consequence on the scalar masses when one probes the symmetry breaking pattern from the weak-scale measurements of the masses [3, 4] (see section 4.1). In fact, we pointed out that the  $D$ -term contribution may arise at the GUT-scale [3], and there have appeared papers which discuss the effect of the  $D$ -term contributions to the Higgs masses in the radiative breaking scenario [20, 21].

In this subsection, we give a simple example which generates a  $D$ -term contribution to the low-energy scalar masses. Take a simple superpotential  $W = h(\phi_1\phi_2 - \mu^2)\chi$ , with  $U(1)$ -charges  $Q = +1$  for  $\phi_1$ ,  $Q = -1$  for  $\phi_2$  and  $Q = 0$  for  $\chi$ . We also have light fields  $z^k$  which couple to the  $U(1)$  gauge fields but do not couple to the heavy fields  $\phi_1$ ,  $\phi_2$  and  $\chi$  in the superpotential. Then the full potential reads as

$$\begin{aligned}
V = & -|F_1|^2 - |F_2|^2 - |F_\chi|^2 - |F^k|^2 - \frac{1}{2}D^2 \\
& + (F_1 h \phi_2 \chi + F_2 h \phi_1 \chi + F_\chi h(\phi_1 \phi_2 - \mu^2) + h.c.) \\
& + gD \left( |\phi_1|^2 - |\phi_2|^2 + \sum_k Q_k |z^k|^2 \right) \\
& + (A h \phi_1 \phi_2 \chi - C h \mu^2 \chi + h.c.) \\
& + m_1^2 |\phi_1|^2 + m_2^2 |\phi_2|^2 + m_\chi^2 |\chi|^2 + m_k^2 |z^k|^2,
\end{aligned} \tag{2.8}$$

where  $F_1$ ,  $F_2$ ,  $F_\chi$ ,  $F^k$  are the auxiliary fields of  $\phi_1$ ,  $\phi_2$ ,  $\chi$ ,  $z^k$ , respectively, and  $D$  is the auxiliary field of the  $U(1)$  gauge field.<sup>5</sup> The SUSY breaking terms  $A$ ,  $C$ ,  $m_\kappa^2$  are  $O(m_S)$ ,  $O(m_S)$ ,  $O(m_S^2)$ , respectively.

We integrate out the heavy fields  $\phi_1$ ,  $\phi_2$  and  $\chi$  from this potential, taking light fields  $z^k$  fluctuating at  $O(m_S)$ . The stationary solutions to the heavy fields are solved by expanding in powers of  $m_S/\mu$ , and one finds

$$\begin{aligned}
\phi_1 = & \mu \\
& + \left[ \frac{-A^2 + C^2}{8h^2\mu} - \frac{m_1^2 + m_2^2}{4h^2\mu} - \frac{m_1^2 - m_2^2}{8g^2\mu} - \frac{\sum_k Q_k |z^k|^2}{4g\mu} \right] + O\left(\frac{m_S^3}{\mu^2}\right),
\end{aligned} \tag{2.9}$$

$$\begin{aligned}
\phi_2 = & \mu \\
& + \left[ \frac{-A^2 + C^2}{8h^2\mu} - \frac{m_1^2 + m_2^2}{4h^2\mu} + \frac{m_1^2 - m_2^2}{8g^2\mu} + \frac{\sum_k Q_k |z^k|^2}{4g\mu} \right] + O\left(\frac{m_S^3}{\mu^2}\right),
\end{aligned} \tag{2.10}$$

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<sup>5</sup>One can also study the same potential after integrating out the auxiliary fields, as one usually does. However we keep the auxiliary fields as independent variables for the later convenience.

$$\chi = -\frac{A-C}{2h} + O\left(\frac{m_S^3}{\mu^2}\right), \quad (2.11)$$

and for the auxiliary fields,

$$F_1 = h\phi_2\chi = -\frac{(A-C)}{2}\mu + O\left(\frac{m_S^3}{\mu}\right), \quad (2.12)$$

$$F_2 = h\phi_1\chi = -\frac{(A-C)}{2}\mu + O\left(\frac{m_S^3}{\mu}\right), \quad (2.13)$$

$$F_\chi = h(\phi_1\phi_2 - \mu^2) = \frac{-A^2 + C^2}{4h} - \frac{m_1^2 + m_2^2}{2h} + O\left(\frac{m_S^3}{\mu}\right), \quad (2.14)$$

$$D = g\left(|\phi_1|^2 - |\phi_2|^2 + \sum_k Q_k |z^k|^2\right) = -\frac{m_1^2 - m_2^2}{2g} + O\left(\frac{m_S^3}{\mu}\right). \quad (2.15)$$

The low-energy effective Lagrangian can be obtained by plugging these solutions into the original Lagrangian, giving

$$\begin{aligned} V_{eff} = & (z^k\text{-independent terms of } O(m_S^2\mu^2)) \\ & + m_k^2 |z^k|^2 - \frac{1}{2}(m_1^2 - m_2^2) \sum_i Q_i |z^i|^2 \\ & + (\text{terms of } O(m_S^5/\mu)). \end{aligned} \quad (2.16)$$

The latter terms are the contributions from the non-vanishing value of the  $D$ -term, giving rise to different masses for different quantum numbers. One also sees that the  $D$ -term contributions vanish if the scalar masses are universal, consistent with the analysis in Ref. [9] reviewed in the previous subsection.

The  $D$ -term contribution exists in general when the rank of the gauge group is reduced by the symmetry breaking [11], and the SUSY breaking terms are non-universal. They give rise to observable effects at the weak scale.

## 2.4 Instability of the Hierarchy

The non-universal SUSY breaking terms have a dramatic consequence when there is a fine-tuning to keep light fields at the weak scale. The light fields acquire masses of the order of the intermediate scale in general. It seems to us that this problem is not widely recognized in the literature (see however [18, 22]).

In the SUSY standard model, we need (at least) two Higgs doublets with opposite hypercharges and these doublet scalars form a SUSY-breaking mixing mass term. Indeed the problem we now discuss is related to the fine-tuning problem of the Higgs doublet mass (the gauge hierarchy problem), which is inevitable in a wide class of SUSY GUT models in order to obtain the light Higgs doublets of the SUSY standard model. We will explain the problem in some detail. The discussions given below are based on the observation in an unpublished work [23].

To exemplify the problem, let us consider the minimal SUSY  $SU(5)$  model [1] whose superpotential is

$$W = \lambda \text{tr} \Sigma^3 + M_\Sigma \text{tr} \Sigma^2 + H_u (f \Sigma + M_H) H_d. \quad (2.17)$$

Here  $\Sigma$ ,  $H_u$  and  $H_d$  are the fields of **24**, **5** and  $\bar{\mathbf{5}}$  representations of  $SU(5)$ , respectively.  $\lambda$ ,  $f$  are dimensionless coupling constants, while  $M_\Sigma$ ,  $M_H$  are GUT scale mass parameters. If we add the SUSY breaking terms arbitrarily, the potential reads

$$\begin{aligned} V = & \left| \frac{\partial W}{\partial \Sigma} \right|^2 + \left| \frac{\partial W}{\partial H_u} \right|^2 + \left| \frac{\partial W}{\partial H_d} \right|^2 + g^2 \sum_\alpha \left( \Sigma^\dagger [T^\alpha, \Sigma] + H_u^\dagger T^\alpha H_u + H_d^\dagger (-T^{\alpha*}) H_d \right)^2 \\ & + \{ \lambda A_\Sigma \text{tr} \Sigma^3 + B_\Sigma M_\Sigma \text{tr} \Sigma^2 + f A_H H_u \Sigma H_d + B_H M_H H_u H_d \\ & + h.c. \}, \end{aligned} \quad (2.18)$$

where  $A_\Sigma$ ,  $B_\Sigma$ ,  $A_H$  and  $B_H$  are the SUSY breaking parameters of order  $m_S$  and we have omitted the SUSY breaking scalar masses which are irrelevant to the following discussions. The  $SU(5)$  gauge coupling constant is denoted by  $g$ . Taking

$$\Sigma = \frac{1}{\sqrt{60}} \begin{pmatrix} 2 & & & & \\ & 2 & & & \\ & & 2 & & \\ & & & -3 & \\ & & & & -3 \end{pmatrix} \sigma, \quad (2.19)$$

the minimal of the potential is located at  $\sigma_0 = \frac{2\sqrt{60}}{3\lambda} M_\Sigma$  in the SUSY limit, which is shifted by  $\delta\sigma = \frac{\sqrt{60}}{3\lambda} (A_\Sigma - B_\Sigma)$  in the presence of the SUSY breaking terms. The mixing mass of the two doublet Higgs bosons  $m_{12}^2 H_u H_d$  is given by

$$m_{12}^2 = \frac{3f}{\sqrt{60}} \sigma_0 (A_\Sigma - B_\Sigma - A_H + B_H) + O(m_S^2), \quad (2.20)$$

where we have used that the supersymmetric mass of the Higgs doublets is fine-tuned to be

$$M_H - \frac{3g}{\sqrt{60}} \sigma_0 = O(m_S) \quad (2.21)$$

at the SUSY limit. Clearly for a class of the SUSY breaking parameters where the combination  $A_\Sigma - B_\Sigma - A_H + B_H$  does not vanish,  $m_{12}^2$  lies at an intermediate scale  $\sim m_S M_X$  and the gauge hierarchy is destabilized.

A crucial observation is, however, that the mixing mass  $m_{12}^2$  becomes of order  $m_S^2$  if we assume the form of the soft terms as in Eq. (2.7). Indeed for the special form of the SUSY breaking terms

$$AW + B_1 \Sigma \frac{\partial W}{\partial \Sigma} + B_2 H_u \frac{\partial W}{\partial H_u} + B_3 H_d \frac{\partial W}{\partial H_d}, \quad (2.22)$$



which follows from the ansatz in section 2.2, the coefficients  $A_\Sigma$  *etc.* in Eq. (2.18) are written as

$$A_\Sigma = A + 3B_1, \quad (2.23)$$

$$B_\Sigma = A + 2B_1, \quad (2.24)$$

$$A_H = A + B_1 + B_2 + B_3, \quad (2.25)$$

$$B_H = A + B_2 + B_3. \quad (2.26)$$

Then we find

$$A_\Sigma - B_\Sigma - A_H + B_H = 0, \quad (2.27)$$

which guarantees the lightness of the Higgs doublets in the minimal  $SU(5)$  model.

### 3 Scalar Potential in the Effective Theory

In the previous section, we have examined some examples to demonstrate that the non-universal SUSY breaking terms can lead to important consequences. In this section, we will give a more general discussion. We consider a softly-broken supersymmetric unified theory<sup>6</sup> whose gauge group is broken at an energy scale  $M_X$ . We will derive the scalar potential  $V_{eff}$  in the low-energy effective theory by integrating out the heavy sector.

#### 3.1 General Discussion

We first list the basic assumptions in the following discussion.

1. The unified theory is described as a renormalizable supersymmetric theory with soft SUSY breaking terms.
2. The unified gauge symmetry is broken at a scale  $M_X$  which is much higher than the SUSY breaking scale  $m_S$  (of  $O(1)$  TeV).
3. SUSY is not spontaneously broken in the absence of the soft SUSY breaking terms in the Lagrangian.
4. All the particles can be classified as heavy (with mass  $O(M_X)$ ) or light (with mass  $O(m_S)$ ) in the absence of the SUSY breaking terms.
5. The light scalar fields have vacuum expectation values (VEVs) as well as fluctuations only of  $O(m_S)$ .

The assumptions here are basically the same as in Ref. [9].

We solve the stationary conditions of the potential of the full theory for the heavy scalar fields while keeping the light scalar fields arbitrary. We then “integrate out” the heavy fields by inserting the solutions of the stationary conditions into the potential.

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<sup>6</sup>The gauge group of the theory is not necessarily grand-unified into a simple group.

The potential obtained in this way should be regarded as the potential of the low-energy effective theory renormalized at the scale  $M_X$ . In deriving the effective scalar potential, we fully utilize the equalities from the gauge invariance given in Appendix A.

The procedure to obtain the low-energy effective potential in this paper is quite similar to that of Ref. [9], though they started from the (minimal) supergravity Lagrangian whereas our starting point is the softly-broken global SUSY. As for the SUSY breaking, we consider a non-universal form which is expected to be realized in realistic models as we discussed in the last section.

### 3.1.1 SUSY Limit

First we review the basic properties of the scalar potential in the absence of the SUSY breaking terms.

The superpotential is a holomorphic function of chiral superfields  $\Phi$

$$W = W(\Phi^\kappa). \quad (3.1)$$

The supersymmetric scalar potential is given as

$$V_{SUSY} = -\frac{1}{2}(D^\alpha)^2 - F_\kappa^* F^\kappa + D^\alpha(z^\dagger T^\alpha z) + F_\kappa^* \frac{\partial W^*}{\partial z_\kappa^*} + \frac{\partial W}{\partial z^\kappa} F^\kappa, \quad (3.2)$$

where  $z^\kappa$  are the scalar components of the chiral superfields  $\Phi^\kappa$ .  $F^\kappa$  is the auxiliary field of  $\Phi^\kappa$ ,  $T^\alpha$  stands for a gauge generator and  $D^\alpha$  is its  $D$ -term.  $z^\dagger T^\alpha z$  is an abbreviation of  $z_\lambda^* (T^\alpha)^\lambda_\kappa z^\kappa$ . Summation over  $\alpha, \kappa$  and  $\lambda$  is implied.

The stationary conditions of the potential are simply

$$F_\kappa^* = \frac{\partial W}{\partial z^\kappa} = 0, \quad (3.3)$$

$$D^\alpha = z^\dagger T^\alpha z = 0 \quad (3.4)$$

under the condition that SUSY is unbroken. We denote the solutions to the stationary conditions as  $z^\kappa = z_0^\kappa$ .

We can always take a basis of  $z^\kappa$  to diagonalize the fermion mass matrix

$$\mu_{\kappa\lambda} = \left. \frac{\partial^2 W}{\partial z^\kappa \partial z^\lambda} \right|_{z=z_0}. \quad (3.5)$$

Then the scalar fields are classified [9] either as “heavy” fields  $z^K, z^L, \dots$ , “light” fields  $z^k, z^l, \dots$ , or Nambu–Goldstone fields which will be discussed just below.

The mass matrix of the gauge bosons is

$$(M_V^2)_{\alpha\beta} = z_{0\lambda}^* \{T^\alpha, T^\beta\}^\lambda_\kappa z_0^\kappa = 2(z_0^\dagger T^\alpha)_\kappa (T^\beta z_0)^\kappa, \quad (3.6)$$

where we used Eq. (3.4) in the last equality, and it can be diagonalized so that the gauge generators are classified into “heavy” (those broken at  $M_X$ )  $T^A, T^B, \dots$  and “light” (which

remain unbroken above  $m_S$ )  $T^a, T^b, \dots$ . For the heavy generators, the fields  $(T^A z_0)^\kappa$  correspond to the direction of the Nambu–Goldstone fields in the field space, which span a vector space with the same dimension as the number of heavy generators. We can take a basis of the Nambu–Goldstone multiplets,  $z^A, z^B, \dots$  so that

$$\sqrt{2}(T^A z_0)^B = M_V^{AB}. \quad (3.7)$$

Here the Nambu–Goldstone fields are taken to be orthogonal to the heavy and light fields such as  $(T^A z_0)^K = 0$ ,  $(T^A z_0)^k = 0$ . Also the vanishing of the  $D$ -terms Eq.(3.4) implies

$$z_0^A = 0. \quad (3.8)$$

### 3.1.2 Soft SUSY Breaking Terms

The soft SUSY breaking terms can be classified by spurion insertions as<sup>7</sup>

$$\int d^2\theta (m_S \theta^2) U(\Phi) = m_S U(z), \quad (3.9)$$

$$\int d^4\theta (m_S \theta^2) B(\Phi, \Phi^\dagger) = m_S F_\lambda^* \frac{\partial B}{\partial z_\lambda^*}, \quad (3.10)$$

$$\int d^4\theta (m_S^2 \theta^2 \bar{\theta}^2) C(\Phi, \Phi^\dagger) = m_S^2 C(z, z^*). \quad (3.11)$$

$U(\Phi)$  is a holomorphic function of the chiral superfields.  $B$  is expressed as

$$B(\Phi, \Phi^\dagger) = B_{2\lambda}^\kappa \Phi_\kappa^\dagger \Phi^\lambda \quad (3.12)$$

for renormalizable theories and thus  $\partial B / \partial z_\lambda^*$  is a function of  $z$ , not of  $z^*$ . A term which depends only on  $\Phi^\dagger$  does not appear in Eq. (3.12), since it can be absorbed into the superpotential as[24]

$$\int d^4\theta (m_S \theta^2) H(\Phi^\dagger) = \int d^2\bar{\theta} m_S H(\Phi^\dagger). \quad (3.13)$$

$C(\Phi, \Phi^\dagger)$  is a bilinear polynomial of the chiral and anti-chiral fields.

The non-renormalization theorem [25] implies that the form  $U(z)$  is preserved from radiative corrections since Eq. (3.9) is an  $F$ -term. On the other hand, the functions  $B$  and  $C$  are generally renormalized as they are the  $D$ -terms. For example, the minimal supergravity induces the soft terms such as  $U = AW(\Phi)$ ,  $B = \Phi^\dagger \Phi$  and  $C = 0$  in the flat limit with  $A$  being a constant. When we take radiative corrections into account,  $U$  remains the same, but  $B$  and  $C$  suffer from the renormalization and in general become non-universal. This observation leads us to investigate non-universal soft terms.

In the rest of this paper, we take the ansatz

$$U(\Phi) = AW(\Phi) \quad (3.14)$$

as in the case of the minimal supergravity, while we admit non-universal structure for  $B$  and  $C$ . We can show that Eq. (3.14) is derived from non-minimal supergravity Lagrangian in the flat limit, provided that the observed sector does not have couplings in the

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<sup>7</sup>We owe this classification to the discussions with K. Inoue, Y. Okada and T. Yanagida.

superpotential to the sector which is responsible for the spontaneous breaking of the local supersymmetry (the “hidden” sector) [26].<sup>8</sup> On phenomenological grounds this “hidden” assumption is widely accepted, since otherwise the large SUSY breaking would directly be transmitted to the observed sector and the SUSY is badly broken in the low-energy effective theory as a consequence. For example, the Yukawa coupling of the “hidden” sector field to the Higgs doublets induces a large mass term of the Higgs bosons of order  $m_S M_{Pl}$  and destabilizes the weak scale. Furthermore we believe that the assumption allows us to integrate the hidden sector first to leave the softly-broken global SUSY theory.

In the last section, we have demonstrated that the fine-tuning of the Higgs doublet masses in the SUSY-GUT is not preserved if the most general soft-terms are switched on, and there arises a large mixing mass term for the scalars. As will be seen later, our ansatz (3.14) avoids the emergence of the dangerous terms from the heavy sector. On the other hand, if we take  $U \neq AW$  the gauge hierarchy achieved by the fine-tuning will be violated by the soft-terms. We will discuss this in section 3.3.

The scalar potential we consider is summarized as follows

$$V = V_{SUSY} + V_{\text{soft}} \quad (3.15)$$

$$\begin{aligned} V_{\text{soft}} = & \{m_S AW(z) + m_S F^\kappa B_\kappa^*(z^*) + h.c.\} \\ & + m_S^2 C(z, z^*). \end{aligned} \quad (3.16)$$

Note that the scalar potential (3.15) is rewritten as

$$\begin{aligned} V = & -F_\kappa^* F^\kappa + \{F^\kappa \frac{\partial W}{\partial z^\kappa} + h.c.\} \\ & - \frac{1}{2} D^\alpha D^\alpha + D^\alpha (z^\dagger T^\alpha z) \\ & + \{m_S AW(z) + m_S B^\kappa(z) \frac{\partial W}{\partial z^\kappa} + h.c.\} \\ & + m_S^2 B_\kappa^*(z^*) B^\kappa(z) + m_S^2 C(z, z^*) \end{aligned} \quad (3.17)$$

by shifting the auxiliary field  $F^\kappa$  as  $F^\kappa \rightarrow F^\kappa + m_S B^\kappa(z)$ . This is the form given at the last section.

## 3.2 Calculation of the Effective Potential

In this subsection we compute the scalar potential of the effective low-energy theory by substituting the heavy fields with the solutions to the stationary conditions of the full potential. For this aim, it is convenient to write both the superpotential and the soft SUSY breaking terms in terms of the variations  $\Delta z^\kappa = z^\kappa - z_0^\kappa$  in place of the scalar fields  $z^\kappa$  themselves. For the renormalizable theories, the superpotential can always be written as

$$W = \frac{1}{2!} \mu_{\kappa\lambda} \Delta z^\kappa \Delta z^\lambda + \frac{1}{3!} h_{\kappa\lambda\mu} \Delta z^\kappa \Delta z^\lambda \Delta z^\mu. \quad (3.18)$$

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<sup>8</sup>Our usage of the term “hidden sector” is somewhat different from that in Ref. [26], where the moduli fields whose  $F$ -components break SUSY are not included in the hidden sector.

The functions  $B^\kappa(z)$ ,  $B_\kappa^*(z^*)$  and  $C(z, z^*)$  are also expanded as

$$B^\kappa(z) = B_1^\kappa + B_{2\lambda}^\kappa \Delta z^\lambda \quad (3.19)$$

$$B_\kappa^*(z^*) = B_{1\kappa}^* + B_{2\kappa}^{*\lambda} \Delta z_\lambda^* \quad (3.20)$$

$$C(z, z^*) = C_{1\kappa} \Delta z^\kappa + C_1^\kappa \Delta z_\kappa^* \quad (3.21)$$

$$+ C_{2\lambda}^\kappa \Delta z_\kappa^* \Delta z^\lambda + \frac{1}{2} C_{2\kappa\lambda} \Delta z^\kappa \Delta z^\lambda + \frac{1}{2} C_2^{\kappa\lambda} \Delta z_\kappa^* \Delta z_\lambda^*. \quad (3.22)$$

From Eq. (3.12), it follows that  $B_1^\kappa = B_{2\lambda}^\kappa z_0^\lambda$ .

The variations of the potential (3.15) with respect to the auxiliary fields  $F$ ,  $D$  and the scalar fields  $z$  are given as

$$\begin{aligned} \frac{\partial V}{\partial F^\kappa} &= -F_\kappa^* + \frac{\partial W}{\partial z^\kappa} + m_S B_\kappa^*(z^*) \\ &= -F_\kappa^* + \mu_{\kappa\lambda} \Delta z^\lambda + \frac{1}{2} h_{\kappa\lambda\mu} \Delta z^\lambda \Delta z^\mu \\ &\quad + m_S (B_{1\kappa}^* + B_{2\kappa}^{*\lambda} \Delta z_\lambda^*), \end{aligned} \quad (3.23)$$

$$\frac{\partial V}{\partial D^\alpha} = -D^\alpha + z^\dagger T^\alpha z, \quad (3.24)$$

$$\begin{aligned} \frac{\partial V}{\partial z^\kappa} &= D^\alpha (z^\dagger T^\alpha)_\kappa + \frac{\partial^2 W}{\partial z^\kappa \partial z^\lambda} F^\lambda \\ &\quad + m_S A \frac{\partial W}{\partial z^\kappa} + m_S \frac{\partial B^\lambda}{\partial z^\kappa} F_\lambda^* + m_S^2 \frac{\partial C}{\partial z^\kappa} \\ &= D^\alpha (z^\dagger T^\alpha)_\kappa + (\mu_{\kappa\lambda} + h_{\kappa\lambda\mu} \Delta z^\mu) F^\lambda \\ &\quad + m_S A (\mu_{\kappa\lambda} \Delta z^\lambda + h_{\kappa\lambda\mu} \Delta z^\lambda \Delta z^\mu) \\ &\quad + m_S B_{2\kappa}^\lambda F_\lambda^* + m_S^2 (C_{1\kappa} + C_{2\kappa}^\lambda \Delta z_\lambda^* + C_{2\kappa\lambda} \Delta z^\lambda) \end{aligned} \quad (3.25)$$

Once the SUSY breaking terms are turned on, the  $F^\kappa$  or  $D^\alpha$  may be non-vanishing, but should be at most  $O(m_S M_X)$  since they have to vanish in the absence of the SUSY breaking terms. We expand the  $F^\kappa$ ,  $D^\alpha$  and  $z^\kappa$  in powers of  $m_S$  such as

$$F^\kappa = F_0^\kappa + \delta F^\kappa + \delta^2 F^\kappa + \dots, \quad (3.26)$$

$$D^\alpha = D_0^\alpha + \delta D^\alpha + \delta^2 D^\alpha + \dots, \quad (3.27)$$

$$z^\kappa = z_0^\kappa + \Delta z = z_0^\kappa + \delta z^\kappa + \delta^2 z^\kappa + \dots, \quad (3.28)$$

with  $\delta^n F^\kappa, \delta^n D^\alpha = O(m_S^n / M_X^{n-2})$  and  $\delta^n z^\kappa = O(m_S^n / M_X^{n-1})$ . Here  $F_0^\kappa$  and  $D_0^\alpha$  are defined as the VEVs in the absence of the SUSY breaking terms and are exactly zero as discussed in section 2.1.1, and the higher order terms are defined as the shifts of their VEVs due to the presence of the SUSY breaking terms. We assume  $z^k = O(m_S)$  for the light fields, *e.g.*,  $z_0^k = O(m_S)$  and  $\delta^2 z^k = \delta^3 z^k = \dots = 0$ .

We can solve the stationary conditions (3.23)–(3.25) by using the above expansions (3.26)–(3.28) order by order. Here we list the equations which are to be used to obtain the scalar potential of the effective theory. For simplicity we assume

$$B_1^k = O(m_S) \quad (3.29)$$

for the time being. This is automatically satisfied if there is no light singlet field. The stationary conditions  $\partial V/\partial F^K = 0$  and  $\partial V/\partial F^A = 0$  imply

$$\delta F_K^* = \mu_{KL}\delta z^L + m_S B_{1K}^* \quad (3.30)$$

and

$$\delta F_A^* = m_S B_{1A}^*, \quad (3.31)$$

$$\begin{aligned} \delta^2 F_A^* &= \frac{1}{2} h_{A\lambda\mu} \delta z^\lambda \delta z^\mu + m_S B_{2A}^\lambda \delta z_\lambda^* \\ &= \frac{1}{2} h_{AKL} \delta z^K \delta z^L + h_{AKk} \delta z^K \delta z^k + m_S B_{2A}^\lambda \delta z_\lambda^* \end{aligned} \quad (3.32)$$

respectively,<sup>9</sup> while  $\partial V/\partial D^A = 0$  gives

$$\delta D^A = (z_0^\dagger T^A)_B \delta z^B + \delta z_B^* (T^A z_0)^B. \quad (3.33)$$

From the conditions  $\partial V/\partial z^K = 0$  and  $\partial V/\partial z^A = 0$ , we find

$$-\mu_{KL} \delta F^L = 0 \quad (3.34)$$

$$\begin{aligned} -\mu_{KL} \delta^2 F^L &= h_{K\lambda\mu} \delta F^\lambda \delta z^\mu + \delta D^A (\delta z^\dagger T^A)_K \\ &\quad + m_S A \mu_{KL} \delta z^L + m_S B_{2K}^\lambda \delta F_\lambda^* + m_S^2 C_{1K}, \end{aligned} \quad (3.35)$$

and

$$\delta D^B (z_0^\dagger T^B)_A = 0, \quad (3.36)$$

$$-\delta^2 D^B (z_0^\dagger T^B)_A = h_{A\lambda\mu} \delta F^\lambda \delta z^\mu + m_S B_{2A}^\lambda \delta F_\lambda^* + m_S^2 C_{1A}, \quad (3.37)$$

respectively.

From Eqs. (3.34) and (3.30), we find the shift of  $z^K$  is

$$\delta z^K = -m_S (\mu^{-1})^{KL} B_{1L}^*. \quad (3.38)$$

On the other hand, Eqs. (3.33) and (3.36) imply

$$\delta z^A = 0. \quad (3.39)$$

Eq. (3.35) gives the solution for  $\delta^2 F^K$  as

$$\delta^2 F^K = \langle F^K \rangle - m_S (\mu^{-1})^{KL} h_{LAl} B_1^A \delta z^l, \quad (3.40)$$

$$\begin{aligned} \langle F^K \rangle &= -(\mu^{-1})^{KL} \{ m_S h_{LAM} B_1^A \delta z^M + m_S A \mu_{LM} \delta z^M \\ &\quad + m_S^2 B_{2L}^A B_{1A}^* + m_S^2 C_{1L} \}. \end{aligned} \quad (3.41)$$

From Eq. (3.37),

$$-\delta^2 D^B (z_0^\dagger T^B)_A = m_S h_{ABK} B_1^B \delta z^K + m_S^2 B_{2A}^B B_{1B}^* + m_S^2 C_{1A}, \quad (3.42)$$

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<sup>9</sup> Gauge invariance implies  $h_{Akl} = O(m_S/M_X)$ , see Appendix A. We have also used  $\delta z^A = 0$ , which will be derived below.

where we have used  $h_{ABk} = O(m_S/M_X)$ , a consequence of the gauge invariance. Note that  $(z_0^\dagger T^B)_A = \frac{1}{\sqrt{2}}(M_V^*)_{BA}$  can be inverted to obtain  $\delta^2 D^B$ . Eq. (3.42) shows that  $\delta^2 D^A$  is a constant independent of the light fields. Therefore we will denote it by  $\langle D^A \rangle$ .

Now it is straightforward to calculate the scalar potential of the low-energy effective theory  $V_{eff}$  by substituting the solutions to the stationary conditions for the heavy fields. The result can be compactly expressed if we define

$$\widetilde{W}(z) = W(z^k, z^K = z_0^K + \delta z^K, z^A = z_0^A + \delta z^A), \quad (3.43)$$

$$\widetilde{B}^k(z) = B^k(z^l, z^K = z_0^K + \delta z^K, z^A = z_0^A + \delta z^A), \quad (3.44)$$

$$\widetilde{C}(z, z^*) = C(z^k, z^K = z_0^K + \delta z^K, z^A = z_0^A + \delta z^A, \dots). \quad (3.45)$$

Note that the above are the functions of only light fields. In particular the  $\widetilde{W}$  is the superpotential of the effective theory. Then we can write down the effective potential as

$$\begin{aligned} V_{eff} = & -F_k^* F^k + F^k \left( \frac{\partial \widetilde{W}}{\partial z^k} + m_S \widetilde{B}_k^* \right) + h.c. \\ & - \frac{1}{2} D^a D^a + D^a (z^\dagger T^a z) \\ & + m_S A \widetilde{W}(z) + h.c. + m_S^2 \widetilde{C}(z, z^*) \\ & + \Delta V, \end{aligned} \quad (3.46)$$

where the new contribution  $\Delta V$  is

$$\begin{aligned} \Delta V = & - \left| \langle F^K \rangle - m_S (\mu^{-1})^{KL} h_{LAk} B_1^A \delta z^k \right|^2 \\ & + \left\{ \langle F^K \rangle \left( \frac{1}{2} h_{K\lambda\mu} \delta z^\lambda \delta z^\mu + m_S B_{2K}^\lambda \delta z_\lambda^* \right) \right. \\ & \quad \left. - m_S (\mu^{-1})^{KL} h_{LAk} B_1^A \delta z^k \left( \frac{1}{2} h_{K\lambda\mu} \delta z^\lambda \delta z^\mu + m_S B_{2K}^\lambda \delta z_\lambda^* \right) \right. \\ & \quad \left. + h.c. \right\} \\ & + \left| m_S B_{1A}^* + m_S B_{2A}^{*K} \delta z_K^* + \frac{1}{2} h_{AKL} \delta z^K \delta z^L \right. \\ & \quad \left. + m_S B_{2A}^{*k} \delta z_k^* + h_{AKl} \delta z^K \delta z^l \right|^2 \\ & + \langle D^A \rangle \delta z^\dagger T^A \delta z. \end{aligned} \quad (3.47)$$

Recall that  $\langle D^A \rangle$  stands for  $\delta^2 D^A$  (see Eq. (3.42)). The last term in Eq. (3.47) comes from the  $D$ -term of the heavy gauge sector and is referred to as the  $D$ -term contribution, while the other contributions are called the  $F$ -term contributions.

Eliminating the auxiliary fields  $F^k$  and  $D^a$  by using the equations of motion, we obtain the SUSY breaking part of the effective potential

$$\begin{aligned} V_{eff, \text{SUSY}} = & m_S A \widetilde{W}(z) + m_S \widetilde{B}^k(z) \frac{\partial \widetilde{W}}{\partial z^k} + h.c. \\ & + m_S^2 \{ \widetilde{B}_k^*(z^*) \widetilde{B}^k(z) + \widetilde{C}(z, z^*) \} + \Delta V. \end{aligned} \quad (3.48)$$

### 3.3 Stability of the Weak Scale

In this subsection, we will investigate whether the effective potential is of the order of magnitude  $m_S^4$ .<sup>10</sup> The occurrence of the terms of  $O(m_S^3 M_X)$  or even larger is very dangerous since it would destabilize the weak scale. Such dangerous terms may appear for a mixing mass term (proportional to  $\delta z^k \delta z^l$ ) and for a linear term (proportional to  $\delta z^k$ ). The latter is related to the notorious difficulty in the presence of the light singlet [27]. Note that the linear term cannot exist for a non-singlet field. Whether the linear term is actually large or not is highly model dependent. We will not discuss this problem further. In the rest of this section we will concentrate on the mixing mass term.

From Eqs. (3.48) and (3.47), it follows that the mixing mass terms are

$$\begin{aligned} & m_S A \mu_{kl} + m_S h_{klm} (B_1^m + B_{2M}^m \delta z^M) \\ & + m_S (B_{2k}^m \mu_{ml} + B_{2l}^m \mu_{mk}) + m_S^2 C_{2kl} + \langle F^K \rangle h_{Kkl} \\ & - m_S (\mu^{-1})^{KL} h_{KAk} B_1^A h_{LMl} \delta z^M - m_S (\mu^{-1})^{KL} h_{KAi} B_1^A h_{LMk} \delta z^M \\ & + m_S h_{KAk} \delta z^K B_{2l}^{*A} + m_S h_{KAi} \delta z^K B_{2k}^{*A}, \end{aligned} \quad (3.49)$$

which are of order  $m_S^2$ . This is due to our assumptions (1)  $U = AW$  and (2)  $B_1^k = O(m_S)$ . We will show that larger terms arise when we relax the assumptions.

First consider the case of  $U \neq AW$ . An inspection similar to the previous subsection shows that there exist terms of order  $m_S M_X$  for the mixing mass

$$\left\langle \frac{\partial^2 U}{\partial z^k \partial z^l} \right\rangle + \langle F^K \rangle h_{Kkl} + O(m_S^2), \quad (3.50)$$

where

$$\langle F^K \rangle = -m_S (\mu^{-1})^{KL} \left\langle \frac{\partial U}{\partial z^L} \right\rangle + O(m_S^2). \quad (3.51)$$

The first term in Eq. (3.50) can be  $O(m_S M_X)$ , since there is no *a priori* reason that the fine-tuning of the Higgs mass achieves simultaneously both in  $W$  and  $U$ . The second term remains large if the conditions  $\partial W / \partial z^K = 0$  and  $\partial U / \partial z^K = 0$  do not hold simultaneously. This observation shows the importance of our ansatz  $U = AW$ .

Next consider the effects of  $B_1^m$  of order  $M_X$ . Obviously  $B_1^m$  is non-zero only for a (light) singlet. Then we find an additional contribution to the mixing mass term

$$m_S B_1^m h_{klm}. \quad (3.52)$$

This again reflects the well-known difficulty related to the light singlet. Indeed a large  $B$  term can appear at the tree-level or may be induced by radiative corrections[27, 28]. To proceed further we need a model dependent analysis which is beyond the scope of the paper.<sup>11</sup>

To conclude, the large mixing mass terms do not arise if (1) one takes the hidden assumption and thus  $U = AW$ , and (2) there is no light singlet.

<sup>10</sup> Here we disregard the constant terms independent of the light fields.

<sup>11</sup> For example, in a flipped  $SU(5)$  model [29], it is known that the light singlet which couples to the Higgs doublets does not induce the large mixing mass.



### 3.4 Mass Terms

We now discuss a *chirality-conserving* mass term, namely the coefficient of  $\delta z^k \delta z_l^*$ . They are easily extracted from Eqs. (3.46) and (3.47), given by

$$\begin{aligned}
& m_S^2 (B_{2k}^m B_{2m}^{*l} + C_{2k}^l) - m_S^2 (\mu^{-1})^{MK} h_{KAk} B_1^A (\mu^{*-1})_{ML} h^{*LBl} B_{1B}^* \\
& - m_S^2 (\mu^{-1})^{KL} h_{KAk} B_1^A B_{2L}^l - m_S^2 (\mu^{*-1})_{KL} h^{*LAk} B_{1B}^* B_{2k}^{*K} \\
& + m_S^2 B_{2k}^A B_A^{*l} + h_{AKk} h^{*ALl} \delta z^K \delta z_L^* \\
& + \langle D^A \rangle (T^A)_k^l.
\end{aligned} \tag{3.53}$$

The term  $m_S^2 (B_{2k}^m B_{2m}^{*l} + C_{2k}^l)$  is present before the heavy sector is integrated out. Therefore it respects the large gauge symmetry of the unified group. On the other hand, other terms coming from  $\Delta V$  can pick up effects of the symmetry breaking.

The last term in Eq. (3.53) is the  $D$ -term contribution. Phenomenologically it is important because it gives an additional contribution to squarks and sleptons [3, 4]. Gauge invariance implies that the non-zero VEV of the  $D$ -term is allowed for the  $U(1)$  factor. Thus it can arise when the rank of the gauge group is reduced by the gauge symmetry breaking. The  $D$ -term contribution is proportional to the charge of the broken  $U(1)$  factor and gives mass splittings within the same multiplet in the full theory.

We can rewrite  $\delta^2 D^A = \langle D^A \rangle$  by using the gauge invariance of  $W$  and  $B$  as

$$\begin{aligned}
\langle D^A \rangle = & -2m_S^2 (M_V^2)^{-1AB} \{ B_{1\kappa}^* (T^B)_\lambda^\kappa B_1^\lambda - B_{1K}^* (T^B)_L^K B_1^L \\
& + C_{2\kappa}^\lambda z_{0\lambda}^* (T^B z_0)^\kappa + C_{2\kappa\lambda} z_0^\lambda (T^B z_0)^\kappa \}.
\end{aligned} \tag{3.54}$$

We will see in the next subsection that the VEV of the  $D$ -term (3.54) vanishes when the SUSY breaking terms are universal and hence the  $D$ -term contribution to the sfermion masses is a characteristic of the non-universal SUSY breaking.

### 3.5 Case of the Universal Soft SUSY Breaking Terms

Let us now discuss the effective potential for the case of the universal soft terms. In addition to our ansatz (3.14), we assume that

$$B(\Phi, \Phi^\dagger) = B\Phi^\dagger\Phi, \tag{3.55}$$

$$C(\Phi, \Phi^\dagger) = C\Phi^\dagger\Phi, \tag{3.56}$$

with dimensionless constants  $B$  and  $C$ . Then we find

$$B_1^\kappa = B z_0^\kappa, \quad B_{2\lambda}^\kappa = B \delta_\lambda^\kappa \tag{3.57}$$

and

$$C_{1\kappa} = C z_{0\kappa}^*, \quad C_1^\kappa = C z_0^\kappa, \quad C_{2\lambda}^\kappa = C \delta_\lambda^\kappa, \quad C_{2\kappa\lambda} = C_{2\kappa}^\lambda = 0. \tag{3.58}$$

In particular,  $B_1^A = C_1^A = 0$  because  $z_0^A = 0$ . Furthermore our assumption  $B_1^k = O(m_S)$  is automatically satisfied since we postulate  $z_0^k = O(m_S)$ , which allows us to use the effective potential (3.46) and (3.47).

We first evaluate  $\Delta V$  in Eq. (3.47). Eq. (3.38) now reads

$$\delta z^K = -m_S(\mu^{-1})^{KL}Bz_{0L}^*. \quad (3.59)$$

The VEVs of the auxiliary fields become

$$\langle F^K \rangle = -m_SA\delta z^K - m_S^2(\mu^{-1})^{KL}Cz_{0L}^*, \quad (3.60)$$

$$\langle D^A \rangle = 2m_S^2(M_V^2)^{-1AB}C(z_0^\dagger T^A z_0) = 0, \quad (3.61)$$

where the last equality is due to Eq. (3.4). Noting that the gauge invariance of the superpotential shows

$$\begin{aligned} h_{BKl}\delta z^K(T^A z_0)^B &= -\mu_{KL}\delta z^L(T^A)_l^K + O(m_S^2) \\ &= m_SB(z_0^\dagger T^A)_l + O(m_S^2) \\ &= 0, \end{aligned} \quad (3.62)$$

we find  $\Delta V$  becomes simply

$$\Delta V = \frac{1}{2}\langle F^K \rangle h_{K\lambda\mu}\delta z^\lambda\delta z^\mu. \quad (3.63)$$

Hence all contributions to the *chirality-conserving* mass terms disappear with the universal soft terms. The SUSY breaking part of the potential is found to be

$$\begin{aligned} V_{eff,SUSY} &= m_SA\widetilde{W} + m_SBz^k\frac{\partial\widetilde{W}}{\partial z^k} + \frac{1}{2}\langle F^K \rangle h_{K\lambda\mu}\delta z^\lambda\delta z^\mu + h.c. \\ &\quad + m_S^2(B^2 + C)z_k^*z^k. \end{aligned} \quad (3.64)$$

An important conclusion is that the scalar mass is common in this case.

To compare our results with those of Ref. [9], we further take

$$B = 1, \quad C = 0, \quad (3.65)$$

as well as  $\mu_{kl} = 0$ ,  $z_0^k = 0$ . Then after a little algebra, we find

$$\begin{aligned} \Delta V &= -\frac{1}{2}m_SAh_{K\lambda\mu}\delta z^K\delta z^\lambda\delta z^\mu \\ &= -m_SA\left(3\widetilde{W} - z^k\frac{\partial\widetilde{W}}{\partial z^k}\right), \end{aligned} \quad (3.66)$$

and the SUSY breaking part is

$$V_{eff,SUSY} = -2m_SA\widetilde{W} + m_S(A+1)z^k\frac{\partial\widetilde{W}}{\partial z^k} + h.c. + m_S^2z_k^*z^k, \quad (3.67)$$

which is in agreement with the result of Ref. [9].

## 4 Phenomenological Implications

In this section, we point out phenomenological implications of the general discussion in the previous sections. The main new feature is that one may have non-universal scalar masses at the GUT scale, both due to the  $D$ -term and  $F$ -term contributions. For first two generation slepton/squark fields, we expect that the superpotential coupling is weak enough to be neglected, and we deal with only the  $D$ -term contributions. It was pointed out in our previous paper [3, 4] that the scalar masses satisfy “sum-rules” corresponding to the symmetry breaking pattern of the grand unified theory, which can be tested at future collider experiments. On the other hand, the Higgs fields and third generation slepton/squark fields are likely to acquire both  $D$ - and  $F$ -term contributions, which may drastically change the analysis of the radiative breaking scenario.

### 4.1 Squark and Slepton Masses

For the first two generation matter fields, the superpotential coupling is small and can be neglected. Then the masses of their scalar components are determined solely by the initial conditions and their gauge quantum numbers. As a consequence, they have a definite pattern in the mass spectrum once one has a specific symmetry breaking pattern from the grand-unified group down to the Standard Model gauge group,  $G_{SM} = SU(3)_C \times SU(2)_L \times U(1)_Y$ .

In our previous paper [3], we showed that the squark and slepton masses satisfy certain “sum rules” for various examples of the symmetry breaking patterns. Let us briefly review the results below, as an example where the  $D$ -term contributions to the scalar masses play a major phenomenological role.

Let us take the following symmetry breaking pattern for instance:

$$SO(10) \xrightarrow{M_U} SU(4)_{PS} \times SU(2)_L \times SU(2)_R \xrightarrow{M_{PS}} G_{SM}. \quad (4.1)$$

There appear  $D$ -term contributions to the scalar masses when the rank of the gauge group reduces from 5 to 4 at the intermediate Pati-Salam symmetry breaking scale  $M_{PS}$ . The matter multiplets belong either to  $L = (\mathbf{4}, \mathbf{2}, \mathbf{1})$  or  $R = (\bar{\mathbf{4}}, \mathbf{1}, \mathbf{2})$  representations, with masses  $m_L^2$  and  $m_R^2$  respectively above  $M_{PS}$ . When the Pati-Salam group breaks to  $G_{SM}$ , we obtain the following masses,

$$m_{\tilde{q}}^2 = m_L^2 + g_4^2 D, \quad (4.2)$$

$$m_{\tilde{u}}^2 = m_R^2 - (g_4^2 - 2g_{2R}^2)D, \quad (4.3)$$

$$m_{\tilde{e}}^2 = m_R^2 + (3g_4^2 - 2g_{2R}^2)D, \quad (4.4)$$

$$m_{\tilde{l}}^2 = m_L^2 - 3g_4^2 D, \quad (4.5)$$

$$m_{\tilde{d}}^2 = m_R^2 - (g_4^2 + 2g_{2R}^2)D, \quad (4.6)$$

where  $D$  represents the  $D$ -term contributions whose normalization is taken arbitrarily. These expressions do not depend on a particular choice of the Higgs representation which

breaks Pati-Salam group, and hence fixed completely by the symmetry breaking pattern.<sup>12</sup> Note that the gauge coupling constants  $g_4^2, g_{2R}^2$  can be determined from the low-energy gauge coupling constants  $\alpha_i(m_Z)$  ( $i = 1, 2, 3$ ) as a function of  $M_{PS}$  alone. On the other hand, one can eliminate  $D, m_L^2$  and  $m_R^2$  from the above formulae, to obtain

$$m_{\tilde{q}}^2(M_{PS}) - m_{\tilde{l}}^2(M_{PS}) = m_{\tilde{e}}^2(M_{PS}) - m_{\tilde{d}}^2(M_{PS}), \quad (4.7)$$

$$g_{2R}^2(M_{PS})(m_{\tilde{q}}^2 - m_{\tilde{l}}^2)(M_{PS}) = g_4^2(M_{PS})(m_{\tilde{u}}^2 - m_{\tilde{d}}^2)(M_{PS}). \quad (4.8)$$

Once we measure the gaugino and scalar masses at low-energy, we can calculate the scalar masses at  $M_{PS}$  as a function of  $M_{PS}$  alone. Since we have two relations for one free parameter  $M_{PS}$ , one can solve for  $M_{PS}$  and further make a consistency check.

There exist more relations when  $SO(10)$  is broken directly into  $G_{SM}$ ,

$$m_{\tilde{q}}^2 = m_{16}^2 + g_{10}^2 D, \quad (4.9)$$

$$m_{\tilde{u}}^2 = m_{16}^2 + g_{10}^2 D, \quad (4.10)$$

$$m_{\tilde{e}}^2 = m_{16}^2 + g_{10}^2 D, \quad (4.11)$$

$$m_{\tilde{l}}^2 = m_{16}^2 - 3g_{10}^2 D, \quad (4.12)$$

$$m_{\tilde{d}}^2 = m_{16}^2 - 3g_{10}^2 D, \quad (4.13)$$

where the only unknown parameters are  $m_{16}^2$  and  $D$  after the measurements of the SUSY breaking masses. We use one relation to fix  $D$ , one for  $m_{16}^2$ , and there remain three relations for the consistency check. On the other hand, we have more free parameters when the symmetry is smaller, for instance,

$$SO(10) \xrightarrow{M_U} SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \xrightarrow{M_{3221}} G_{SM}. \quad (4.14)$$

In this case we cannot even determine the parameters in the original model.

On the other hand, let us consider flipped  $SU(5)$  model [30, 29] as an example of non-unified model. Its gauge group is  $SU(5) \times U(1)$  and is not grand-unified into a single group. First of all, we have two independent gaugino masses  $M_{SU(5)}$  and  $M_{U(1)}$  at the GUT scale, which results in the low-energy gaugino masses which do not necessarily satisfy the GUT-relation. However, one can test the scenario measuring  $M_2$  and  $M_3$  at the weak scale and see whether they unify at the same scale where the gauge coupling constants  $\alpha_2$  and  $\alpha_3$  unify. On the scalar masses, we have three independent masses  $m_{10}^2, m_5^2$  and  $m_1^2$  at the GUT scale, and an unknown  $D$ -term in addition. There are five observable scalar masses, and we know the scale where  $SU(2)$  and  $SU(3)$  coupling constants meet. Therefore we are left with one additional relation which can be checked. The scalar masses satisfy the following relations at  $M_X$  where  $SU(2)$  and  $SU(3)$  unify to  $SU(5)$ ,

$$m_{\tilde{d}}^2(M_X)^2 = m_{10}^2 + \left( -\frac{2}{5}g_{SU(5)}^2 + \frac{1}{40}g_{U(1)}^2 \right) D, \quad (4.15)$$

---

<sup>12</sup>One may anticipate that there are also  $F$ -term contributions since we need a superpotential  $RR\chi$  to generate right-handed neutrino masses where  $\chi$  transforms as  $(\mathbf{15}, \mathbf{1}, \mathbf{3})$ . Indeed, right-handed scalar neutrino may acquire contributions like  $F_\chi \tilde{N}^c \tilde{N}^c$ . However other squark, slepton fields cannot acquire  $F$ -term contribution because of the  $G_{SM}$  invariance.

$$m_{\tilde{l}}(M_X)^2 = m_{\tilde{5}}^2 - \left( \frac{3}{10} g_{SU(5)}^2 + \frac{3}{40} g_{U(1)}^2 \right) D, \quad (4.16)$$

$$m_{\tilde{u}}(M_X)^2 = m_{\tilde{5}}^2 + \left( \frac{1}{5} g_{SU(5)}^2 - \frac{3}{40} g_{U(1)}^2 \right) D, \quad (4.17)$$

$$m_{\tilde{q}}(M_X)^2 = m_{10}^2 + \left( \frac{1}{10} g_{SU(5)}^2 + \frac{1}{40} g_{U(1)}^2 \right) D, \quad (4.18)$$

$$m_{\tilde{e}}(M_X)^2 = m_1^2 + \frac{1}{8} g_{U(1)}^2 D. \quad (4.19)$$

Then the “sum rule” is obtained as

$$m_{\tilde{d}}^2(M_X) - m_{\tilde{l}}^2(M_X) = m_{\tilde{q}}^2(M_X) - m_{\tilde{u}}^2(M_X). \quad (4.20)$$

Though the “sum rules” of the scalar masses are weak when the symmetry is small, one can acquire useful information by combining the scalar mass spectrum with that of the gauginos. We pointed out that the gaugino masses satisfy the so-called GUT-relation

$$\frac{M_1}{\alpha_1} = \frac{M_2}{\alpha_2} = \frac{M_3}{\alpha_3}, \quad (4.21)$$

even when the grand-unified group breaks down to  $G_{SM}$  in several steps. On the other hand, non-GUT models like flipped  $SU(5)$  do not necessarily have a unified gaugino mass, and hence do not predict GUT-relation of the gaugino masses. Therefore, one can draw useful information on the GUT models by measuring the scalar and gaugino masses in future experiments. We present a “score sheet” of various models in Table 1. Here we refer to the paper [31] on superstring predictions.<sup>13</sup>

## 4.2 Radiative Breaking Scenario

There has been reported several remarkable results based on the radiative breaking scenario [14] and the universal scalar mass hypothesis at the GUT scale. One of them is that the LSP should be gaugino dominant to correctly reproduce the weak scale [33], when top quark is heavy. This gives a stringent constraint on the cosmic abundance of the neutralino. The other is that the proton decay in the minimal  $SU(5)$  SUSY-GUT cannot be consistent with the present bound if one further requires that the LSP does not overclose the Universe [34]. Since both analyses crucially depend on the universal scalar mass hypothesis, there may be qualitatively different consequences in the non-minimal case.

The crucial equations to determine  $m_Z$  and  $\tan \beta$  in the minimal SUSY standard model (MSSM) are the following ones at the tree-level,

$$m_Z^2 = -\frac{m_1^2 - m_2^2}{\cos 2\beta} - (m_1^2 + m_2^2 + 2\mu^2), \quad (4.22)$$

---

<sup>13</sup>It is noteworthy that the superstring with dilaton  $F$ -term also leads to the same relation. This is amusing because one needs rather big threshold corrections for the gauge coupling constants to reconcile the difference between the apparent GUT-scale and the string scale [32]. Exactly the same correction appears both for the gauge coupling constants and the gaugino masses to give the same relation as in the (field theoretical) GUT models.

$$\sin 2\beta = -\frac{2m_3^2}{m_1^2 + m_2^2 + 2\mu^2}, \quad (4.23)$$

where  $m_1^2$ ,  $m_2^2$  refer to the soft SUSY breaking part of the Higgs masses,  $m_3^2$  the off-diagonal mass, and  $\mu$  the higgsino mass parameter. When one adopts the universal scalar mass hypothesis, the large top quark Yukawa coupling drives  $m_2^2$  much smaller than  $m_1^2$  in general as far as  $\tan\beta$  is not very large. This gives a too large value of the first term in r.h.s of Eq. (4.22) in general, which should be compensated by the negative contributions of  $\mu^2$ . Of course the details depend on the renormalization group analysis, it can be studied semi-analytically as far as one can neglect the bottom quark Yukawa coupling constant [35]. In this case,

$$m_1^2 = m_t^2, \quad (4.24)$$

$$m_2^2 = m_t^2 - 3I, \quad (4.25)$$

$$m_3^2 = B\mu, \quad (4.26)$$

Here,  $I$  is defined by the following,

$$I = I_{SS}m_\infty^2 + I_{GG}M_\infty^2 + I_{GA}M_\infty A_\infty + I_{AA}A_\infty^2, \quad (4.27)$$

with the coefficients  $I_{SS}$ ,  $I_{GG}$ ,  $I_{GA}$ ,  $I_{AA}$  are functions of the top quark Yukawa coupling only, and  $m_\infty$ ,  $M_\infty$ ,  $A_\infty$  are the universal scalar mass, universal gaugino mass, and universal trilinear coupling at the GUT scale, respectively. One can rewrite the Eqs. (4.22) as

$$\begin{aligned} \mu^2 = & 3\frac{\tan^2\beta}{\tan^2\beta - 1}(I_{SS}m_\infty^2 + I_{GG}M_\infty^2 + I_{GA}M_\infty A_\infty + I_{AA}A_\infty^2) \\ & -(m_\infty^2 + 0.52M_\infty^2) - m_Z^2/2. \end{aligned} \quad (4.28)$$

The coefficient  $I_{GG}$  varies from 0.6 to 1.2, and hence  $\mu^2$  is always an increasing function of  $M_\infty$ . For small  $\tan\beta$ , the first term dominates, and one has a large  $\mu^2$ . For moderately large  $\tan\beta$ , the LEP bound on  $M_2$  gives a lower bound on  $M_\infty$ . Though  $m_\infty^2$  and  $M_\infty A_\infty$  terms may give negative contributions to the above equation, their coefficients are in general not large, and one has to take  $m_\infty$  or  $|A_\infty|$  very large to make higgsino light. Since such parameters are not favored from the naturalness point of view, one reaches the conclusion that the higgsino-like LSP is disfavored in the radiative breaking scenario. This is especially true in the no-scale case, where all  $m_\infty^2$ ,  $M_\infty A_\infty$ , and  $A_\infty^2$  term vanish. It is not possible to obtain a higgsino-like LSP within the no-scale models. Even with non-vanishing  $m_\infty$  and  $A_\infty$ , it was shown that there are no solutions with a higgsino-like LSP [33] after including the one-loop effects on the Higgs potential.

However, the situation drastically changes when one incorporates the possible  $D$ -term contributions to  $m_1^2$  and  $m_2^2$ . Let us imagine the initial condition  $m_1^2 = m_\infty^2 - \Delta m^2$ ,  $m_2^2 = m_\infty^2 + \Delta m^2$ . This gives an extra contribution  $\Delta\mu^2$  to the  $\mu^2$  as

$$\Delta\mu^2 = \frac{1}{2\cos 2\beta}(2 - (1 - \cos 2\beta)I_{SS})\Delta m^2, \quad (4.29)$$

which allows a lighter higgsino compared to the case of the universal scalar mass.

The fact that the lighter higgsino is allowed has a very strong impact in the minimal  $SU(5)$  GUT, where there has been claimed that the nucleon decay via the dimension-five operators cannot be consistent with the longevity of the Universe [34]. The nucleon decay rate is roughly proportional to  $M_\infty/m_\infty^2$  when  $M_\infty \lesssim m_\infty$  [36], while gaugino-like LSP has an abundance proportional to  $m_\infty^2/M_\infty$ . However, the abundance becomes much smaller once the smaller higgsino mass parameter is allowed. Therefore, one can take  $M_\infty$  to be much smaller than  $m_\infty$  without worrying about the LSP abundance, which opens a consistent region between the nucleon decay experiments and the LSP abundance.

### 4.3 FCNC

The assumption of the universal scalar mass is motivated to explain the smallness of the flavor-changing neutral current (FCNC) due to the loops of SUSY particles [7]. Since we have relaxed this assumption, the readers may be worried about FCNC.

There are two classes of non-minimal effects, one which does not break the degeneracy between sfermion masses with the same quantum numbers, the other which does. The non-minimality we discussed in subsection 4.1 could have been generated by the renormalization between the Planck scale and the GUT scale. The implicit assumption here is that the Yukawa interactions are small for first two generations, even to the superheavy fields which are completely decoupled from the low-energy effective action. As far as *all* of their interactions in the superpotential are small, the only renormalization effects arise due to the gauge interactions, and hence universal for different generations. The degeneracy of sfermion masses at the initial condition ensures the degeneracy at the weak scale. In this case, non-minimal nature of the radiative corrections does not break the degeneracy.

On the other hand, there are many source of radiative corrections which could break the degeneracy. For instance, there are attempts to explain the degeneracy of sfermion masses based on horizontal symmetries [37]. If the horizontal symmetry is gauged and breaks spontaneously, however, there may be  $D$ -terms in the horizontal gauge group which potentially breaks the degeneracy again. Another example is when the first two generations also have  $O(1)$  Yukawa interactions beyond the GUT scale. In this case there are two sources of non-degeneracy: (1) renormalization due to the Yukawa interactions, (2)  $F$ -term contributions to the scalar masses when the heavy particles decouple. There are no discussions on these effects in the literature to our knowledge.

It is noteworthy, however, that FCNC can be sufficiently suppressed even with non-degenerate initial condition. If the scalar masses turn out to be relatively smaller than the gaugino masses, radiative corrections between the Planck or GUT scale and the weak scale tend to make squark masses universal due to the gluino contribution. Indeed, SUSY breaking via moduli  $F$ -term condensation in superstring inspired supergravity models give non-universal scalar masses which depend on modular weights, and in principle can lead to large FCNC processes. However, they can be sufficiently suppressed due to the renormalization effects at least for some region of the parameter space [31].

## 4.4 Renormalization Between GUT and Planck Scales

As repeatedly emphasized through the text, one of the important sources of the non-minimality is the renormalization between the GUT and the Planck scales. Let us briefly comment when and how these effects can be important despite the apparent closeness of the two scales. Indeed, most of the analyses in the literature completely ignore the difference of these two scales.

One example is when the constraint is marginal. For instance, it was shown in Refs. [12, 13] that one has an upper bound on the gaugino mass in a restricted class of minimal supergravity model with  $m_\infty = A_\infty = 0$  at the GUT scale. The argument comes from the fact that the right-handed slepton acquires a mass only of

$$m_{\tilde{l}_R}^2 \simeq 0.87 M_1^2, \quad (4.30)$$

and hence is smaller than  $M_1$ . Then there is a danger that  $\tilde{l}_R$  becomes lighter than the lightest neutralino. One is forced to use mixing in the neutralino sector to push the LSP mass down from  $M_1$ . (Higgsino LSP does not exist as a solution in the radiative breaking scenario with the universal scalar masses.) However mixing can be substantial only when gaugino mass is close to  $m_Z$ , and one obtains an upper bound on the gaugino mass. It was translated to an upper bound on the slepton mass,  $\lesssim 150$  GeV [12, 13]. The situation completely changes when one includes the running of the slepton mass between Planck and GUT scales. Then one obtains

$$m_{\tilde{l}_R}^2 \simeq 3.1 M_1^2, \quad (4.31)$$

and one does not need a substantial mixing in the neutralino sector any more. Therefore the upper bound on the gaugino mass becomes obsolete due to this effect.

Another example is when there is a relatively large coupling constant. If we require  $m_b$ - $m_\tau$  mass relation, one generally needs top quark Yukawa coupling constant of  $\simeq 2.0$  at the GUT scale for small  $\tan\beta$ , and  $\simeq 0.8$  for large  $\tan\beta \simeq 60$ . Let us take minimal  $SU(5)$  for clarify our discussions. Then  $H_u$  has much smaller  $m^2$  at the GUT scale compared to  $H_d$  even when they start from the same value at the Planck scale. One has to go through the following analyses. First one solves the renormalization group running of the Higgs masses for **24**, **5**,  $\bar{\mathbf{5}}$ , and also other SUSY breaking parameters. Then employ the formulae presented in section 3 to integrate out the superheavy fields to obtain the low-energy effective action. Such a non-minimality may change the parameter space of the radiative breaking scenario substantially, however does not affect FCNC constraint since the effects of  $h_t$  does not appear in the first- and second-generation scalar masses.

## 5 Conclusions

In this paper, we have derived the low-energy effective Lagrangian in the scalar sector starting from a unified theory with non-universal soft SUSY breaking terms. Such non-universal soft terms arise if we take a flat limit of the supergravity where the Kähler



potential is a non-minimal one. One should note that this is indeed the case in the string-inspired model where the moduli fields are responsible for the SUSY breaking. Even if the soft terms have the universal structure at the gravitational scale, they get renormalized and as a result become non-universal in general when the energy scale goes down to the GUT scale. Therefore we expect that the soft terms are non-universal at the GUT scale and it is important to investigate its consequences at low energies.

We have calculated the scalar potential of the low-energy theory by explicitly integrating out the heavy sector. The SUSY breaking part of the scalar potential is summarized in Eqs. (3.48) and (3.47). We found some new contributions to the soft terms which can be non-zero only when the soft terms of the full theory are non-universal. In particular, the sizable  $D$ -term contributions generally exist in the chirality conserving scalar masses when the rank of the gauge group is reduced by the gauge symmetry breaking. Its phenomenological implications were discussed in our previous papers [3, 4]. Another important point is concerned with the gauge hierarchy problem. Many of the SUSY GUT models achieve the small Higgs doublet mass by a fine-tuning of the parameters in the superpotential. If the soft terms are turned on, however, a SUSY breaking Higgs mass term can become heavy and the weak scale will be destabilized. We showed that all mass terms remain at the weak scale if the soft terms are restricted to those derived from the supergravity model where the hidden sector decouples from the observable sector in the superpotential.

We have also discussed other phenomenological implications. Recall that there are many sources which give the non-universal scalar masses, including the  $D$ -term and/or  $F$ -term contributions discussed in this paper. This non-universality changes the predictions of the radiative electroweak symmetry breaking, usually assuming the common mass for the two Higgs doublet bosons. In particular, the higgsino can be the dominant component of the LSP even when the top quark is heavy: if we assume the universal scalar mass the LSP is dominated by a gaugino component. This cures the apparent conflict of the nucleon life time and the LSP relic abundance. In the no-scale model, on the other hand, we pointed out that the upper bound on the right-handed slepton mass [12, 13] disappear if we properly incorporate the renormalization group flow between the GUT scale and the Planck scale. Further study of the radiative breaking scenario without the universal scalar mass hypothesis should be encouraged.

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## A Equalities from gauge invariance

We summarize the consequence of the gauge invariance of the superpotential  $W$ .

From the gauge invariance of  $W$ , there is the equality

$$\frac{\partial W}{\partial z^\kappa} (T^\alpha)_\lambda^\kappa z^\lambda = 0, \quad (\text{A.1})$$

where summation over repeated indices  $\kappa$  and  $\lambda$  is understood. If we differentiate the above equality with respect to  $z^\mu$ , we obtain

$$\frac{\partial^2 W}{\partial z^\kappa \partial z^\mu} (T^\alpha)_\lambda^\kappa z^\lambda + \frac{\partial W}{\partial z^\kappa} (T^\alpha)_\mu^\kappa = 0. \quad (\text{A.2})$$

By further differentiating the above, one finds

$$\frac{\partial^3 W}{\partial z^\kappa \partial z^\mu \partial z^\nu} (T^\alpha)_\lambda^\kappa z^\lambda + \frac{\partial^2 W}{\partial z^\kappa \partial z^\nu} (T^\alpha)_\mu^\kappa + \frac{\partial^2 W}{\partial z^\kappa \partial z^\mu} (T^\alpha)_\nu^\kappa = 0. \quad (\text{A.3})$$

The equalities (A.2) and (A.3) become

$$\mu_{AB} = 0 \quad (\text{A.4})$$

$$h_{\kappa\mu\nu} (T^\alpha)_\lambda^\kappa z_0^\lambda + \mu_{\kappa\nu} (T^\alpha)_\mu^\kappa + \mu_{\kappa\mu} (T^\alpha)_\nu^\kappa = O(m_S) \quad (\text{A.5})$$

respectively, in the case of  $W$  of Eq. (3.18). Eq. (A.5) can be written as

$$h_{AKL} (T^B z_0)^A + \mu_{MK} (T^B)_L^M + \mu_{ML} (T^B)_K^M = O(m_S) \quad (\text{A.6})$$

$$h_{ACL} (T^B z_0)^A + \mu_{MK} (T^B)_C^M = O(m_S) \quad (\text{A.7})$$

$$h_{AKk} (T^B z_0)^A + \mu_{MK} (T^B)_k^M = O(m_S) \quad (\text{A.8})$$

$$h_{Akl} (T^B z_0)^A = O(m_S) \quad (\text{A.9})$$

$$h_{ACk} (T^B z_0)^A = O(m_S) \quad (\text{A.10})$$

in terms of components.

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	$\alpha_i$	$M_i/\alpha_i$	$m_i^2$
$SU(5) \rightarrow G_{SM}$	natural	common	testable
$SO(10) \rightarrow G_{SM}$	natural	common	testable
$SO(10) \rightarrow G_{PS} \rightarrow G_{SM}$	adjustable	common	testable
$SO(10) \rightarrow G_{3221} \rightarrow G_{SM}$	adjustable	common	not testable
$SO(10) \rightarrow G_{3211} \rightarrow G_{SM}$	adjustable	common	not testable
$SU(5) \times U(1) \rightarrow G_{SM}$	adjustable	common only for $i = 2, 3$	testable
superstring with dilaton $F$ -term	adjustable	common	testable
superstring with moduli $F$ -term	adjustable	not common	not testable

Table 1: The “score sheet” how well we can distinguish between various models. The intermediate groups are defined as  $G_{PS} = SU(4)_{PS} \times SU(2)_L \times SU(2)_R$ ,  $G_{3221} = SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ ,  $G_{3211} = SU(3)_C \times SU(2)_L \times U(1)_R \times U(1)_{B-L}$ . The row  $\alpha_i$  refers to the unification of the gauge coupling constants, where “natural” means that the unification is automatic, while “adjustable” employs either particular particle content or threshold corrections to reproduce the observed gauge coupling constants. The row  $M_i/\alpha_i$  refers to the gaugino masses. The row  $m_i^2$  states whether the model predicts a definite pattern which is testable using the low-energy scalar mass spectrum.